

Inhomogeneous reheating scenario with low scale inflation and/or MSSM flat directions

Marieke Postma

The Abdus Salam ICTP, Strada Costiera 11, Trieste, Italy

Abstract

We discuss the constraints on the inhomogeneous reheating scenario. In particular, we discuss the prospects for low scale inflation with a Hubble constant of the order of the gravitino mass, and the possibility that an MSSM flat direction is responsible for the density fluctuations. Thermal effects are generically fatal for the scenario, and can only be avoided for small enough mass and couplings of the flat direction field, excluding MSSM flat directions. Prompt decay at the end of inflation bypasses thermal constraints, and is compatible with both low scale inflation and MSSM flat directions. However, the level of non-Gaussianity is acceptably small only for a small window of inflaton mass and couplings. The upshot is that tuning of parameters is needed for the inhomogeneous reheating scenario to work.

1 Introduction

The inflationary paradigm in which the inflaton sector is responsible for the density perturbations is economical. One potential serves many purposes: driving inflation, ending inflation, and generating the observed spectrum of density perturbations. The resulting models of inflation are predictive, and falsifiable by experiment. The flip side of the coin, though, is that they are restrictive too, and it has proven extremely hard to build realistic models of inflation. Often a considerable amount of fine-tuning is needed for the model to satisfy all constraints. For this reason it may be worthwhile to explore alternatives.

Inflationary models can be “liberated” if its task list is reduced [1]. This is the idea behind both the curvaton scenario [2, 3] and the inhomogeneous reheating scenario [4], in which not the inflaton field but some other field is responsible for the density perturbations. The inflaton sector merely serves to drive and end inflation, and is considerably less constraint. Of course, the price to be paid is that a new field has to be introduced into the theory. But the costs are minimized if the new field and/or the new scales introduced are already present in our models of particle physics. There is already an extensive literature exploring this possibility in the context of the curvature scenario [5, 6], but little has been done in the context of the inhomogeneous reheating scenario (but see [7, 8]). In this paper we will concentrate on the inhomogeneous reheating scenario. In particular, we will address the following two questions. Is low scale inflation — with the Hubble scale during inflation of the order of the gravitino mass — possible, such that the inflaton sector can be naturally identified with SUSY breaking sector? Can any of the flat directions in the minimal supersymmetric standard model (MSSM) be responsible for the density fluctuations?

Low scale inflation is hard to realize in the conventional setting in which the inflaton is responsible for the density fluctuations.¹ In that case the density fluctuations are proportional to H_*/M_{pl} , with the subscript $*$ denoting the quantity evaluated at the time observable scales leave the horizon, some 60 e-folds before the end of inflation. Unless the slow roll parameter $\epsilon = (M_{\text{pl}}V'/V)^2 \ll 1$, which often requires some amount of fine tuning, high scale inflation is needed to get the observed size of density perturbations. In the curvaton scenario the density perturbations are proportional to H_*/σ_* with σ the vacuum expectation value (VEV) of the curvaton field. Since the VEV can be much lower than the Planck scale, low scale inflation is more natural in the curvaton scenario. However, in [10] it was shown that for the scenario to work the scale of inflation cannot be too low: $H_* \gtrsim 10^7 \text{ GeV}$.

In the inhomogeneous reheating scenario the density perturbations are proportional to H/S_* or H/M , depending on the form of the decay rate, where S is the VEV of the flat direction field responsible for the density perturbations and M is some cutoff scale. Since both M and S_* can be much smaller than the Planck scale,

¹The exception are “new inflation” models, in which the inflaton starts out close to the origin [9]. When Taylor expanded around the inflaton VEV, the linear term is anomalous small, making this different behaviour possible.

also here, low scale inflation seems natural. We will see, that indeed, low scale inflation with H_* of the order of the gravitino mass is possible in this context, although only in a small part of parameter space. The fluctuating decay rate scenario can then be married with any of the interesting inflaton models, in which the inflaton originates from the SUSY breaking sector [11].

In both the curvaton and inhomogeneous reheating scenario a scalar field other than the inflaton is responsible for the density perturbations. This scalar has to be light compared with the scale of inflation to be able to fluctuate freely, and produce the scale invariant perturbation spectrum observed. Obvious candidates for this scalar field are the MSSM flat directions. Within the curvaton scenario this possibility has been studied in [6]. It was found that only under very special conditions can the MSSM scalar produce the density perturbations. The main problem is that the curvaton has to (nearly) dominate the energy density before decay. This means large initial VEVs and thus potential problems with non-renormalizable operators, and this means late decay and thus potential problems with disastrous finite temperature effects. In this respect, the inhomogeneous reheating scenario offers better prospects, as the MSSM flat direction does not need to dominate the energy density at the time of decay. Consequently, its VEV can be small enough to avoid problems with the non-renormalizable potential, and the density perturbations may be generated before thermal effects become important.

This paper is organized as follows. In the next section, we provide the background material. We describe those features of the inflaton and MSSM sector important for the inhomogeneous reheating scenario. In section 3 we turn to the description of the various constraints on the inhomogeneous reheating scenario. Section 4 discusses the various time scales in the problem, giving further insights in the nature of especially the thermal constraints. We address the prospects for model building, concentrating on the possibility of low scale inflation and using MSSM flat directions, for a polynomial decay rate in section 5, while section 6 discusses the same issues in the context of an (approximate) constant decay rate. In section 7 we discuss the varying mass scenario, which is a variation of the inhomogeneous reheating scenario, and point out its virtues and drawbacks for model building. Finally, we conclude in section 8.

2 Preliminaries

The idea behind the inhomogeneous reheating scenario is the following. Consider the decay of the inflaton field, or more generally the field dominating the energy density (FDED), into standard model degrees of freedom. Suppose now that the decay rate for this process depends on the vacuum expectation value of some field. In supersymmetric theories as well as in superstring inspired theories, the effective couplings are functions of the various fields in the theory, and thus so is the decay rate. If one of these fields, call it S , is light during inflation it can condense with

a large VEV. Moreover, it fluctuates freely during inflation. As a result, the decay rate is spatially fluctuating on superhorizon scales.

FDED decay will happen at slightly different times in different parts of the universe. The regions in which decay has taken place are filled with radiation, while the not yet decayed regions are matter dominated. The universe expands at a different rate in different regions, resulting in fluctuations in the reheat temperature, hence, to adiabatic density perturbations.

2.1 Decay rates

The decay rate of the field dominating the energy density can be schematically written as $\Gamma = \lambda^2 m K$, with m the FDED mass, λ its coupling, and K a phase space factor. All three quantities can have a field dependence, leading to different realizations of the inhomogeneous reheating scenario. The various possibilities will be discussed in detail in later sections. For now we just want to remark that we can divide the decay rates into two general classes, which differ in their S -dependence. Here and in the following S denotes the flat direction (“flaton”) field responsible for the density perturbations.

The first class consists of decay rates polynomial in S , of the form

$$\Gamma_\phi = \Gamma_0 S^p, \quad (p \geq 1), \quad (1)$$

Decay rates of this form can arise from non-normalizable operators. If there are several fields S_i that can give rise to a decay rate of the above form, the dominant contribution will come from the field with the largest VEV. A large VEV favors directions with a small mass. This may naturally select a flaton with $m_S \lesssim 0.1 H_*$, leading to a scale invariant spectrum.

The second class of decay rates are of the form

$$\Gamma_\phi = \Gamma_0 \left[1 + \left(\frac{S}{M} \right)^q \right]^r, \quad (q \geq 1). \quad (2)$$

with $S < M$. This is the $p = 0$ limit of the polynomial decay rate. At zeroth order the decay rate is constant, the S dependence comes only in through higher order terms. Such decay rates may be obtained from normalizable operators, or from phase space effects. We will refer to the above decay rates as respectively “polynomial” and (approximately) “constant”.

2.2 Inflaton sector

We will not be concerned with the specific origin of inflation. All that is needed is that there is period of inflation, long enough to solve the horizon and flatness problem, and with a Hubble constant that is almost constant $|\dot{H}_*/H_*^2| \ll 1$. However, the mass m_ϕ of the field that stores most of the energy at the end of inflation will be an important parameter, so we digress somewhat on that.

In one-field models of inflation the inflaton mass is bounded by the Hubble scale during inflation: $m_\phi \lesssim H_*$. Since the density perturbations are not produced by the inflaton field, the slow roll parameter η does not need to be much smaller than unity. Fast roll inflation is possible for $m_\phi \gtrsim H_*$, but the number of e-folds is negligible small unless $m_\phi \rightarrow H_*$.

There is more freedom in multiple field models of inflation. Consider for example hybrid inflation with a potential [12]

$$V(\phi, \chi) = V_0 + \delta V(\chi) - \frac{1}{2}m_\phi^2\phi^2 + \frac{1}{2}\lambda'\chi^2\phi^2 + \frac{1}{4}\lambda\phi^4. \quad (3)$$

In supersymmetric theories the couplings λ' and λ are related. If the inflaton χ is responsible for the density perturbations, λ' has to be sufficiently small to assure a scale invariant spectrum, but this requirement can be relaxed in the inhomogeneous reheating scenario. Inflation occurs in the regime $\chi > \chi_c = m_\phi/\sqrt{\lambda'}$, and the inflaton rolls slowly in the potential $\delta V(\chi)$. Inflation ends when $\chi < \chi_c$, and ϕ acquires a VEV

$$\langle\phi\rangle = \frac{2V_0^{1/2}}{m_\phi}, \quad \text{and} \quad \lambda = \frac{4V_0}{\langle\phi\rangle^4} = \frac{m_\phi^4}{4V_0}, \quad (4)$$

where we have set $V = 0$ in the vacuum. The field dominating the energy density at the end of inflation is the waterfall field ϕ . Taking $\lambda \sim 1$, the mass of this field in the true vacuum is $m_\phi \sim V_0^{1/4}$ and thus $m_\phi^2 \gg H_*^2 \sim V_0$.² Another example of two-field inflation is the recently proposed “new old inflation” [13]. The potential is of the hybrid inflation type, and also in this case $m_\phi \gg H_*$ is possible.

In the next sections the mass m_ϕ denotes the mass of the field dominating the energy density at the end of inflation, i.e., the inflaton field in one field models of inflation and the waterfall field in hybrid inflation. With an abuse of language will refer to this field in both cases as the inflaton field. We parametrize

$$\begin{aligned} m_\phi &\sim \beta H_*, & (1\text{FI}), \\ m_\phi &\sim \beta\sqrt{H_*}, & (2\text{FI}), \end{aligned} \quad (5)$$

with $\beta \lesssim 1$ for both one field inflation (1FI) and two field inflation (2FI) such as hybrid inflation. Since it is not expected that $m_\phi \gg V_0$, the β -factor is maximum for 2FI.

A variation of the inhomogeneous reheating scenario was proposed in [7]. Suppose that the inflaton decays into heavy particles ψ , whose mass is set by a flat direction VEV. The particles freeze-out, and when they become non-relativistic they soon come to dominate the energy density in the universe. The density perturbations are generated during the decay of the ψ particles, and are sourced in this

²We will use units in which the reduced Planck mass $M_{\text{Pl}} = 8\pi G = 1$

case by a varying mass as opposed to a varying coupling.³ This scenario seems less economical since it needs the introduction of yet another field. We consider it for completeness though. The mass m_ψ is bounded by the temperature $m_\psi \lesssim T$ for thermal production. If they are direct inflaton decay products, $m_\psi < m_\phi$ for perturbative decay, and $m_\psi < 10^2 - 10^4 m_\phi$ for bosons and fermions respectively in non-perturbative decay [14].

We will only consider perturbative inflaton decay. It is well known that the waterfall field in hybrid inflation generically decays non-perturbatively, in a rapid process dubbed instant preheating [15]. The negative (mass)² at the end of inflation leads to a spinodal instability, and the ϕ condensate breaks up almost instantly. All long wavelength modes below some critical wavelength k_* are excited. The waterfall field still dominates the energy density, but now the energy is not only stored in the zero-mode but in all modes with $k \lesssim k_*$. If decay happens before annihilation reactions become important, the inhomogeneous reheating scenario still works as before.

2.3 Low scale inflation

Several inflationary models have been constructed in which the inflaton sector is linked to SUSY breaking [11]. Then naturally $V_0^{1/4} \sim 10^{-8}$ and $H_* \sim V_0^{1/2} \sim m_{3/2} \sim 10^{-16}$ for gravity mediated SUSY breaking. In anomaly mediated SUSY breaking schemes, the scale of inflation $V_0^{1/4}$ is larger by one or two orders of magnitude, whereas for gauge mediation the scale is smaller.

2.4 MSSM flat directions

The flat directions of the MSSM consist of gauge invariant operators composed of MSSM scalar fields [16]. This polynomial is commonly parametrized as $X = \phi^n$, with n the dimension of X . A non-zero VEV for ϕ will break the standard model gauge symmetry. All fields entering in the flat directions which are left after the Higgs mechanism (except the linear combination which receives the VEV after diagonalization) have masses of order $h\phi$, with h a gauge coupling, due to their D -term couplings to the flat direction VEV [17]. In addition superpotential couplings of the form $W = h\phi qq$ lead to effective mass terms $M_q \sim h\phi$ with h the MSSM Yukawas.

Therefore, if the flaton S responsible for the density perturbations is identified with one of the MSSM flatons ϕ , at least one of its couplings is of gauge strength $h \sim 0.1$. The Yukawas vary between $10^{-6} < h < 1$.

³Of course, one could also consider a decay rate for ψ with a varying coupling constant. This is the same as considering inflaton decay with a varying coupling, only less economical, since an extra field is introduced.

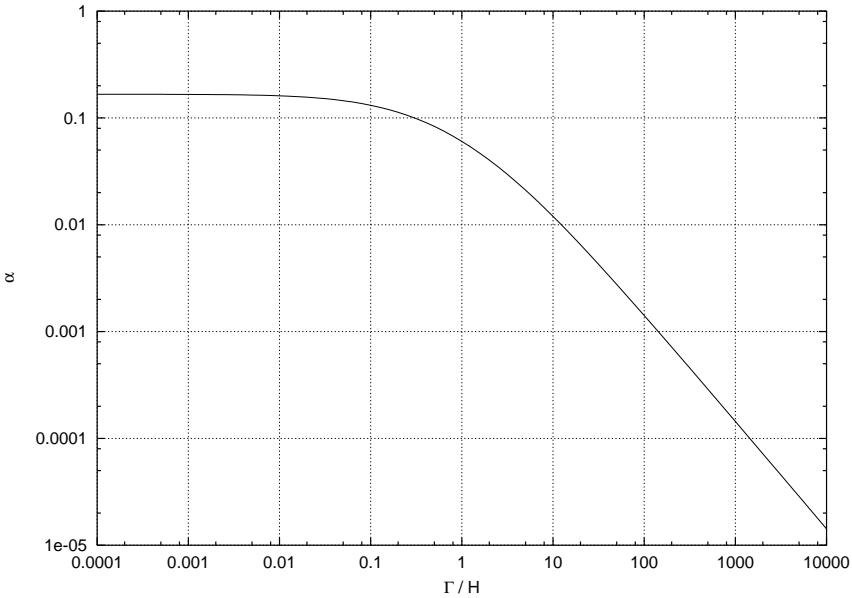


Figure 1: Efficiency parameter α as a function of $\Gamma_\phi/H_{\text{end}}$

3 Constraints

We list here the various constraints on the inhomogeneous reheating scenario.

3.1 Density perturbations

The density perturbations can be parametrized by the gauge invariant quantity ζ , which describes the density perturbations on uniform curvature slices [18]. The perturbation generated by a fluctuating inflaton decay rate are [4, 7, 19]

$$\zeta = \alpha \frac{\delta \Gamma_\phi}{\Gamma_\phi}, \quad (6)$$

with α the efficiency parameter. Γ_ϕ is to be evaluated at the time of decay, when $H \sim \Gamma_\phi$. After reheating is completed the metric can be written as $ds^2 = -dt^2 + g^2(\Gamma_\phi)tdx^2$ [20]. The function $g \propto \Gamma_\phi^{-\alpha}$ parametrizes the difference in expansion rates in different regions, and thus the resulting density perturbations. It can be calculated numerically by integrating the coupled equations of motion for the inflaton and the radiation bath, together with the Friedman equation. The slope gives the efficiency parameter α , which is shown in Fig. 1.

The parameter α depends on the ratio $\Gamma_\phi/H_{\text{end}}$ with H_{end} the Hubble constant at the end of inflation. If $\Gamma_\phi \gg H_{\text{end}}$, decay occurs almost instantaneously in all regions of space, almost independently of the fluctuations in the decay rate, and the resulting density perturbations will be small. In the opposite limit $\Gamma_\phi \ll H_{\text{end}}$ the efficiency is maximal, and α reaches its maximum value $\alpha = 1/6$, which agrees with

analytical estimates [4, 7, 19]. We can parametrize

$$\alpha \approx \begin{cases} 1/6, & \Gamma_\phi \lesssim 10^{-2} H_{\text{end}}, \\ 1/6 - 1/15, & \Gamma_\phi \lesssim 10^{-2} - 1 H_{\text{end}}, \\ 0.1(H_{\text{end}}/\Gamma_\phi), & \Gamma_\phi \gtrsim H_{\text{end}}. \end{cases} \quad (7)$$

The value for ζ measured by WMAP is $\zeta = 2 \times 10^{-5}$ [21]. The right amount of density perturbations for a polynomial decay rate of the form Eq. (1) are obtained if

$$\frac{\delta S}{S} \Big|_{\text{dec}} = \frac{2 \times 10^{-5}}{\alpha p}, \quad (8)$$

The equivalent for the constant decay rate of Eq. (2) is

$$\frac{S^{q-1} \delta S}{M^q} \Big|_{\text{dec}} = \frac{2 \times 10^{-5}}{\alpha q r}, \quad (9)$$

where we have used $S/M \ll 1$.

If the density perturbations are produced during the decay by a field ψ other than the inflaton field, that dominates the energy density sometime after the end of inflation, then generically $\Gamma_\psi \ll H_{\text{end}}$ and $\alpha = 1/6$. If the FDED has a varying mass, density perturbations are already produced before decay, as domination happens at different times in different parts of the universe. The resulting perturbations after decay are⁴

$$\zeta = \frac{1}{6} \frac{\delta \Gamma_\psi}{\Gamma_\psi} - \frac{1}{3} \frac{\delta m_\psi}{m_\psi}, \quad (10)$$

to be evaluated at the time of decay. If the mass $m_\psi \propto S$, the decay rate is polynomial and the density perturbations are up to a minus sign given by Eq. (8) with $p = 1$ and $\alpha = 1/6$.

In the standard scenario in which the inflaton is responsible for the density perturbations the non-Gaussianity is small. The slow roll conditions assure that the perturbations are Gaussian at production, and as the adiabatic perturbations remain constant on superhorizon scales, no further non-Gaussianity is produced. In contrast, large non-Gaussianity is possible in the inhomogeneous reheating scenario. Apart from the possibility of non-Gaussianity at production, non-Gaussianities can be produced due to the non-linear evolution if S starts rolling in the potential, due to the non-linear relation between the decay rate and δS , and due to the non-linear relation between the decay rate and the final perturbations [7, 20, 23]. The general rule is that the less efficient the transfer of S perturbations in metric perturbations is, the larger the non-Gaussianities. This is easy to understand, since if the transfer is less efficient, larger fluctuations $\delta S/S$ are needed to obtain the observed spectrum.

⁴While finishing up this paper [22] appeared, claiming the density perturbations produced in the fluctuating mass case is about a factor 10 larger than the estimate Eq. (6), which is taken from [7]. The difference will not affect our conclusions in an essential way.

A level of Gaussianity as probed by WMAP is assured if

$$\delta S \lesssim 10S, \quad (11)$$

both at the time of production as well as at decay. In particular, this means $\delta S_* \approx H_*/(2\pi) \lesssim 10S_*$. The transfer of perturbations is inefficient if the transfer parameter α is small, or for the constant decay rate if the ratio S/M is small. Indeed, combining Eqs. (8, 9) with the constraint from Gaussianity, Eq. (11), bounds $\alpha \gtrsim 10^{-4}$ and $\alpha(S_{\text{dec}}/M)^q \gtrsim 10^{-4}$ respectively. When $V''(S_*) \lesssim H^2$ the flaton zero mode and its evolution starts rolling in its potential. This may introduce additional inefficiencies, which will be discussed in the next subsection.

The produced spectrum of perturbations is nearly scale invariant, as dictated by observations, if the mass of the fluctuating field is sufficiently small $m_S \lesssim 0.1H_*$. Note that the same must hold for the effective mass $m_{\text{eff}} = V''$ generated by higher order terms in the potential. A quartic term $V = \kappa|S|^4$ will lift the flatness of the potential, unless κ is exceedingly small. For this reason we will not consider such potentials in the following. One should also consider non-renormalizable operators in the potential of the form [24]

$$V_{\text{NR}} = \frac{|\kappa|^2 |S|^{2(n-1)}}{\Lambda^{2(n-3)}}, \quad (12)$$

with Λ some cutoff scale, e.g. the Planck mass. In an expanding universe supersymmetry is broken dynamically, leading to soft mass terms for all scalars of the order of the Hubble constant. Such large masses $m \sim H$ will spoil the scale invariance of the perturbations, and should be avoided. We will assume that the Hubble induced flaton mass is at least one order of magnitude below its canonical value, assuring it is unimportant at all times. Soft mass terms can be suppressed by either invoking symmetries, or by allowing tuning.

The density perturbations produced through the inhomogeneous reheating mechanism are by assumption the dominant ones; other sources should lead to negligible perturbations. In particular, the perturbations produced by the inflaton field should be negligible small, which is assured for $H_* \lesssim H_{\text{max}} \equiv 10^{-6}$. Further, at the time of S -decay $\rho_S/\rho_{\text{total}} \ll 1$, otherwise the flaton perturbations will dominate through the curvaton mechanism. In the curvaton scenario $\zeta \approx r/(2\pi)H_*/S_*$, with $r = \rho_S/\rho_{\text{total}} \sim S_*^2 g^{-1}$ and g the largest flaton coupling, determining its decay rate. The curvaton perturbations are sub-dominant if $g \gg S_*^2$.

3.2 Evolution of S and δS

The decay rate is a function of the flat direction field, and therefore a function of time. The inflaton decays when $H/\Gamma_\phi(S)$ drops below unity. This excludes cases in which the decay rate decreases faster than the Hubble constant.

Once the effective flaton mass becomes of the order of the Hubble rate, $V''(S_*) \sim H^2$, the flaton starts oscillating in its potential. For a quadratic potential $\rho_S \sim a^{-3}$.

During inflaton oscillations, the scale factor of the universe scales as $a \propto H^{-2/3}$, and $S \propto H$. The ratio $H/\Gamma_\phi(S)$ remains constant, and the inflaton does not decay. If the potential is dominated by non-renormalizable operators, then S rolls down approximately critically damped $V''_{\text{NR}} \sim H^2$. We parametrize the post-inflationary evolution of the flaton field

$$S \equiv f S_*, \quad (13)$$

with

$$f \propto \begin{cases} H, & (V_{m_S}), \\ H^{\frac{1}{n-2}}, & (V_{\text{NR}}), \end{cases} \quad (14)$$

where $V_{m_S} = (1/2)m_S^2|S|^2$ and V_{NR} given by Eq. (12).

Further, the inflaton should decay before the decay of the flat direction field: $\Gamma_\phi > \Gamma_S$. This is automatically satisfied if $\Gamma_\phi > m_S$ and flaton couplings less than unity. The same constraints hold if it is not the inflaton but some other field dominating the energy density, which has a varying decay rate.

As the Hubble constant drops below $\sqrt{V''(S)}$ not only the zero mode but also the fluctuations δS start evolving. The equations of motion for the zero mode and the superhorizon fluctuations are

$$\begin{aligned} \ddot{S} + 3H\dot{S} + V'(S) &= 0, \\ \ddot{\delta S} + 3H\dot{\delta S} + V''(S)\delta S &= 0. \end{aligned} \quad (15)$$

For a quadratic potential, the equations of motion for S and δS are identical (to linear order) and the ratio $(\delta S/S)$ remains constant. If the potential is dominated by non-renormalizable operators the fluctuations will be damped with respect to the zero-mode according to [25]

$$D \propto H^{\frac{n-4}{2(n-2)}}, \quad (16)$$

with the damping factor D defined as

$$\left(\frac{\delta S}{S}\right) \equiv D \left(\frac{\delta S}{S}\right)_*. \quad (17)$$

There is no damping for $n = 4$ operators.

Using the definitions of f and D we can express the equations for the density perturbations Eqs. (8, 9) in terms of the quantities during inflation:

$$S_* \approx 8 \times 10^3 \alpha p D H_*, \quad (18)$$

for a polynomial decay rate. Here we have used $\delta S_* \approx H_*/(2\pi)$. Gaussianity as in Eq. (11) requires $D \gtrsim 10^{-3} p^{-1} ((1/6)/\alpha)$. The equivalent expression for a constant decay rate is

$$\frac{H_* S_*^{q-1}}{M^q} \approx \frac{10^{-4}}{\alpha q r D f^q} \quad (19)$$

with $D f^q \gtrsim 10^{-3} (M/S_*)^q ((1/6)/\alpha) (qr)^{-1}$ to assure Gaussianity.

Apart from the zero temperature potential the evolution of S and δS can be sourced by finite temperature effects.

3.3 Thermal constraints

The radiation bath affects the flaton condensate in two ways, through thermal scattering and through thermal corrections to the flaton mass. The fields that couple to the flat directions through a gauge/Yukawa coupling h , denote them by χ , have a large effective mass $m_\chi \sim hS$. The χ particles are in thermal equilibrium with the radiation bath, if their effective mass is smaller than the temperature

$$m_\chi \sim hS \lesssim T. \quad (20)$$

It is important to realize that even before inflaton decay has completed, there is a dilute plasma with temperature [26]

$$T = (T_R^2 H)^{1/4} = (\Gamma_\phi H)^{1/4}, \quad (21)$$

with $T_R \approx \sqrt{\Gamma_\phi}$ the reheat temperature at the end of the reheating process when $\Gamma_\phi \sim H$. In between the end of inflation and inflaton decay the effective temperature scales as $T \propto H^{1/4}$, instead of the $T \propto H^{1/2}$ during radiation domination. The plasma reaches its maximum temperature T_{\max} immediately after the end of inflation:

$$T_{\max} \sim \left(\frac{H_{\text{end}}}{\Gamma_\phi} \right)^{1/4} T_R \sim (H_{\text{end}} \Gamma_\phi)^{1/4}, \quad (22)$$

with H_{end} the Hubble constant at the end of inflation.

The back reaction of the χ quanta on the flat direction field induces thermal corrections to the flaton mass. If the χ quanta are in thermal equilibrium, when Eq. (20) is satisfied, there is an induced “plasma mass” of the form

$$\delta m_{\text{pl}}^2 \sim h^2 \frac{n_\chi}{E_\chi} \sim h^2 T^2, \quad (hS < T), \quad (23)$$

where the last equality hold if the χ quanta have a thermal distribution with $n_\chi \sim T^3$ and $E_\chi \sim T$. There is also a thermal correction in the opposite limit — $T \lesssim m_\chi$, and the χ quanta are out of equilibrium — as a result of integrating out the heavy degrees of freedom [27, 28]. The massless vector and chiral superfields, i.e., those not “Higgsed” by the flat direction VEV S , generate a two-loop free energy proportional to $h'^2 T^4$, with h' the corresponding gauge or Yukawa coupling. The running of h' changes at the renormalization scale $\mu \sim m_\chi \sim hS$, if m_χ is charged under the gauge group, respectively couples to the matter field in question. Integrating out the heavy χ fields above this scale, leads to an effective “RG potential” of the form:

$$\delta V_{\text{RG}} \sim c_T h'^4 T^4 \log \left(\frac{hS}{T} \right)^2, \quad (hS > T), \quad (24)$$

with c_T a constant of order one which can have either positive or negative sign, depending on the matter content. The sign is negative if the running is dominated by integrating out Higgsed gauge bosons.

Large thermal masses, $m_{\text{th}} > H > m_S$, induce early oscillations of the flat direction field. During these oscillations, the energy density stored in the flat direction scales as [29]

$$\rho_S(H) = \frac{m(H)}{m(H_0)} \frac{H^2}{H_0^2} \rho_S(H_0). \quad (25)$$

Thermal evaporation of the flat direction through collisions with the χ particles occurs if (1) the scattering rate is large $\Gamma_{\text{scat}} \gtrsim H$, (2) the χ particles are in thermal equilibrium, and (3) the energy density stored in the thermal bath is larger than the energy density in the flat direction. This last condition will generically be satisfied. The equilibrium condition is given by Eq. (20). The scattering rate is $\Gamma_{\text{scat}} \sim n_\chi \sigma$. For a superpotential term $W = hS\chi\chi$ the cross section for χS scattering is $\sigma \sim h^2 \alpha / E_{\text{cm}}^2$, where we have assumed scattering is dominated by fermions. The typical center of mass energy is $E_{\text{cm}} \sim \sqrt{Tm}$, the mean of the typical χ energy ($\sim T$) and S energy ($\sim m = hT$, where the last equality holds when thermal masses are important). Then $\Gamma_{\text{scat}} \sim hg^2T$, with $g \sim 0.1$ a gauge coupling [27].

Early thermal evaporation obviously kills the inhomogeneous reheating scenario. As we will see, early induced oscillations are generically also fatal. Therefore, for the fluctuating decay scenario to work, thermal effects should be negligible. There are three possibilities. The first is that the particles coupling to the flat directions are out of equilibrium. Thermal scattering is negligible. There is a renormalization group (RG) induced potential of the form Eq (24). If the thermal mass is less than the Hubble scale $\delta V''_{\text{RG}} \ll H^2$, the fields remains frozen and thermal effects are negligible.

The second possibility is that the effective plasmon mass is lower than the temperature. Thermal scattering is delayed with respect to the onset of induced oscillations. Therefore, thermal effects are absent if the induced plasma mass, Eq (23), is smaller than the Hubble rate.

The time when thermal effects become important may be delayed if the thermal plasma is initially far from a thermal equilibrium distribution. If $n_\chi/E_\chi \ll T^2$, the plasma mass is then below its equilibrium value. The scattering rate may also be suppressed, as it scales with n_χ .⁵ Likewise, if the χ are non-relativistic, the RG potential can be suppressed if the free energy of the massless field are below their equilibrium value.

An initial distribution with $n_\chi/E_\chi \ll T^2$ can happen if the plasmons are direct inflaton decay products with $E_\chi \sim m_\phi \gg T$. Or else, if the plasmons are not inflaton decay products, their initial number density is small. The plasmons acquire chemical equilibrium when the rate for number changing interactions Γ_{int} becomes of the order of the Hubble constant. $\Gamma_{\text{int}} \sim h_{\text{pl}}^2 g^2 T$, where h_{pl} is the coupling of χ to the thermal bath and g a gauge coupling. If the plasmons carry gauge charges, $h_{\text{pl}} \sim g \sim 10^{-1}$; the time scale for chemical equilibrium $\Gamma_{\text{int}} \sim 10^{-4}T$ is smaller than

⁵Note that there is an enhancement in the scattering rate for smaller induced mass, as $\sigma \propto 1/m_{\text{th}}$.

the plasma mass hT , and the onset of plasma induced oscillations can be delayed only for $h \gtrsim 10^{-4}$.

The induced thermal masses and scattering rates are a function of the temperature $T \sim (H\Gamma_\phi(S))^{1/4}$. Hence, the constraints depend on how the inflaton decay rate depends on S , and are therefore model dependent. The dependence of the plasma temperature on H , Eq. (21), is derived for a constant inflaton decay rate. We will approximate the temperature by this formula also in the case of a variable decay rate, so that the temperature becomes a function of both H and S .

If oscillations of the flat direction field are set off by thermally induced masses, the field amplitude decreases as given by Eq. (25). For plasma masses $m_{\text{pl}} \sim hT$ this implies

$$f \propto H^{7/(8+p)}, \quad (26)$$

with $p = 0$ for a constant decay rate. Since the potential is approximately quadratic $D \sim 1$. We postpone a discussion of f and D for RG induced potentials to the next section.

To avoid gravitino overproduction and thereby spoiling the successful nucleosynthesis predictions, the reheat temperature has to be sufficiently small $T_R \lesssim 10^{-10}$, or equivalently $\Gamma \lesssim 10^{-19}$ [30]. This is not a hard bound, in the sense that depending on the specifics of the SUSY breaking mechanism, the bound can shift some orders of magnitude. Moreover, the gravitino problem can be solved for example by invoking a period of thermal inflation [31]. A bound that cannot be tampered with is the big bang nucleosynthesis (BBN) bound: The field dominating the energy density should decay before BBN, $T_R \gtrsim 10^{-22}$ or $\Gamma \gtrsim 10^{-43}$.

In this context, we should also remark that the baryon asymmetry of the universe has to be created either during or after FDED decay. If FDED decay happens below the electroweak scale $\Gamma_\phi \lesssim 10^{-33}$, sphalerons are out of equilibrium, and an Affleck-Dine like mechanism for baryogenesis is required.

4 Time scales

The important time scales in the problem are the following. Inflation ends at

$$H_{\text{end}} \sim \zeta H_*, \quad (27)$$

with ζ parameterizing the difference between the time observable scales leave the horizon, and the end of inflation. For simplicity we set $\zeta \sim 1$, although lower values are possible. For example, in 1FI with a quartic potential $\zeta \sim 0.1$.

The large VEV of the inflaton gives a large mass to its decay products, and inflaton decay is kinematically forbidden during inflation. Decay becomes possible as soon as the inflaton starts oscillating in its potential. Therefore, the Hubble constant at the time of decay is $H_{\text{dec}} \sim \min[\Gamma_\phi, H_{\text{end}}]$.

The scales at which the vacuum mass and effective mass generated by non-renormalizable operators become important are respectively $H \sim m_S$ and

$$H_{\text{NR}} \sim \frac{\kappa S^{n-2}}{\Lambda^{n-3}}. \quad (28)$$

We denote the scale at which thermal effects become important by H_{th} . The particles coupling to the flat direction are in equilibrium for $T > hS$, or equivalently $H > H_{\text{eq}}$, with

$$H_{\text{eq}} \sim \min \left[\frac{h^4 S^4}{\Gamma_\phi}, H_{\text{end}} \right]. \quad (29)$$

Then for $H > H_{\text{eq}}$, $H_{\text{th}} \sim H_{\text{pl}}$ the time scale at which the plasma mass of Eq. (23) becomes of the order of the Hubble scale, while for $H < H_{\text{eq}}$, $H_{\text{th}} \sim H_{\text{RG}}$ the time scale at which the RG potential of Eq. (24) becomes important. Here

$$H_{\text{pl}} \sim \min \left[h^{4/3} \Gamma_\phi^{1/3}, H_{\text{end}} \right], \quad (30)$$

$$H_{\text{RG}} \sim \min \left[\frac{h^4 \Gamma_\phi}{S^2} \left(2p + 1 + p(p-1) \log \left(\frac{hS}{T} \right) \right), H_{\text{end}} \right], \quad (31)$$

where $p = 0$ corresponds to the constant decay rate. As discussed in section 3, evaporation is delayed with respect to the onset of thermally induced oscillations. The onset of thermal effects can be delayed if the initial plasma is far from an equilibrium distribution, and the number densities of the particles giving mass to S is less than the equilibrium value $n_\chi \ll T^3$. The delay is until number changing interactions become important, at $H_{\text{del}} \sim \Gamma_{\text{int}}$ with

$$H_{\text{del}} \sim \min \left[(h_{\text{pl}} g^2)^{4/3} \Gamma_\phi^{1/3}, H_{\text{end}} \right], \quad (32)$$

with h_{pl} the coupling of the particle responsible for the induced thermal mass of S , i.e., the plasmon χ which couples to S if the χ 's are relativistic, and the light degrees of freedom whose free energy is altered by the running of χ for non-relativistic χ 's.

We can then distinguish the following cases:

1. $\Gamma_\phi \gtrsim H_{\text{end}}$. Inflaton decay is kinematically inaccessible during inflation, and the inflaton decays promptly at the end of inflation.
2. The inflaton decays after the end of inflation, but while the flaton is still frozen, i.e., before the onset of oscillations of the flat direction field. Consequently, there is no damping $D = 1$ and $S_{\text{dec}} = S_*$. This is the case for

$$H_{\text{end}} > \Gamma_\phi > m_S, H_{\text{th}}, H_{\text{NR}}. \quad (33)$$

3. Inflaton decay occurs after the end of inflation, and after the flaton field has started rolling in the quadratic potential, the non-renormalizable potential and/or the thermally induced potential. Damping and the evolution of S should be taken into account, and f and D are generically different from unity.

Case 1 Case 1 — prompt decay — is obtained if $\Gamma_\phi \gtrsim H_{\text{end}}$. The transfer of flaton perturbations to metric perturbations is inefficient in this limit, and large non-Gaussianity is possible. Gaussianity as in Eq. (11) and the requirement of prompt decay together constrain the decay rate, or equivalently the parameter Γ_0 (see Eqs. (1, 2)), to the range

$$(5p \times 10^2 H_*)^{-p} \lesssim \frac{\Gamma_0}{H_*} \lesssim 5p \times 10^2 H_*^{-p}, \quad (\Gamma_\phi \text{ pol.}) \quad (34)$$

$$1 \lesssim \frac{\Gamma_0}{H_*} \lesssim 5qr \times 10^2 \left(\frac{H_*}{M} \right)^q, \quad (\Gamma_\phi \text{ cnst.}) \quad (35)$$

for a polynomial and constant decay rate respectively. Here we have used Eqs. (18, 19). There is only a small window of Γ_0 values for which prompt decay is possible.

There are no thermal or damping effects, and the only constraints come from the observed magnitude of perturbations Eqs. (18, 19) with $f = D = 1$, and scale invariance $m_S, \sqrt{V''_{\text{NR}}(S_*)} \lesssim 0.1 H_*$. In addition the gravitino problem has to be addressed for $H_* \gtrsim 10^{-19}$.

Case 2 Inflaton decay occurs after the end of inflation, but before the Hubble constant drops below the effective mass of the flat direction field, which aside from the vacuum mass can have contributions from thermal effects and from non-renormalizable operators in the Lagrangian. The zero-mode and fluctuations are frozen, and $f = D = 1$.

The flaton vacuum mass m_S , respectively the effective mass from non-renormalizable operators, is smaller than the Hubble constant at inflaton decay for

$$H_{\text{dec}} \gtrsim m_S, H_{\text{NR}}. \quad (36)$$

Thermal effects play no rôle if the plasmons are in equilibrium, but inflaton decay happens before the plasma mass becomes important, that is if

$$H_{\text{dec}} > H_{\text{pl}}, H_{\text{eq}} \quad (37)$$

It should be understood that $H_{\text{pl}}, H_{\text{eq}} < H_{\text{end}}$, i.e., plasma masses are not important immediately after inflation has ended, nor are the plasmons out of equilibrium from the start. Another possibility for the thermal effects to be absent is that the plasmons fall out of equilibrium before the equilibrium thermal effects become important. This includes the case in which the plasmons are out of equilibrium right from the start. A RG correction to the potential is generated, which should be negligible $H_{\text{RG}} < H_{\text{dec}}$. Both conditions are satisfied for

$$\begin{aligned} H_{\text{eq}} &> H_{\text{pl}}, H_{\text{dec}}, \\ h &\lesssim S^{1/2}. \end{aligned} \quad (38)$$

Finally, thermal effects may not set in until $H \sim H_{\text{del}}$, if the plasma is initially far from the thermal equilibrium distribution. Thermal effects are avoided if

$$H_{\text{del}} \lesssim H_{\text{dec}} \quad (39)$$

Case 3 The flaton starts rolling in its potential before inflaton decay. A polynomial decay rate changes with changing S , whereas a constant decay rate remains (approximately) constant. Hence, these two cases are quite different and we will discuss them separately.

As said, the decay rate is time-dependent for a polynomial decay rate. The first remark to be made is that the expression for the density perturbations Eq. (6) is derived for a constant decay rate. However, we expect that the time-dependence will not significantly alter the size of the final perturbations.

Since the decay rate changes with time, there is the possibility that the decay rate decreases faster than the Hubble constant, and inflaton decay will not occur. In fact, this is the case if the flaton starts oscillating in the quadratic potential, when $H \lesssim m_S$. The flaton red shifts $S \propto H$, see Eq. (14), and decay is impossible for all $p \geq 1$. Similarly, decay does not occur when the flaton starts rolling in the non-renormalizable potential, and $n \leq p + 2$. If oscillations of the flat direction field are set off by a plasma mass $m_{\text{pl}} \sim hT$, the field amplitude decreases as given by Eq. (26). For $p \geq 2$ the decay rate decreases faster than the Hubble rate, and the inflaton cannot decay during thermally induced oscillations either. Therefore

$$\Gamma_\phi > m_S, H_{\text{th}, p \geq 2}^{\text{eq}}, H_{\text{NR}, p \geq n+2} \quad (40)$$

should be satisfied for the inhomogeneous reheating scenario to be possible. Here the superscript eq indicates that the plasmons coupling to the flat direction are in thermal equilibrium, and the subscripts $p \geq 2$ and $p \geq n + 2$ indicate that only decay rates satisfying these conditions are implied.

For the same Hubble constant during inflation, the flat direction VEV at inflaton decay is smaller (larger) than in case 1-2, due to both the damping of the fluctuations and the evolution of S . Now $S = fS_* = 8 \times 10^3 \alpha p f D H_*$, with $\alpha \approx 1/6$. For $fD < 1$ ($fD > 1$), inflaton decay occurs for Hubble constants which are a factor $(fD)^p$ smaller (larger) compared to case 2. The bounds on the flaton mass and couplings, Eqs. (36, 37, 38, 40), are stronger (weaker) by factors of $(fD)^p$. In addition, there is the new constraint that the damping should not be too large, $D \gtrsim 10^{-3}$, to assure a Gaussian perturbation spectrum.

Consider first the effect of the non-renormalizable potential V_{NR} of Eq. (12) with $n > p + 2$, so that inflaton decay is possible. If $H_{\text{NR}} > \Gamma_\phi$, the flat direction field starts slow rolling in the potential before the epoch of inflaton decay. For simplicity, we will concentrate on the case that thermal effects are negligible at all times $H_{\text{NR}} > \Gamma_\phi > m_S, H_{\text{th}}$. Then $fD < 1$ and all constraints are stronger. The decay rate at $H < H_{\text{NR}}$ is

$$\Gamma_\phi(H) = \left(\frac{H}{H_{\text{NR}}} \right)^{2/(n-2)} \Gamma_\phi(H_{\text{NR}}) \quad (41)$$

Decay occurs at $\Gamma_\phi(H_{\text{dec}}) \sim H_{\text{dec}}$. The damping factor is $D = (H_{\text{dec}}/H_{\text{NR}})^{(n-4)/2(n-2)}$. The amplitude S_* can be found as a function of H_* by solving the equation $S_* \approx$

$10^3 p D H_*$. The amplitude at the time of inflaton decay is $S_{\text{dec}} \sim (H_{\text{dec}}/H_{\text{NR}})^{1/(n-2)} S_*$. For example, for $p = 2$ and taking $\kappa = 1$ and $\Lambda = 1$, this leads to

$$\begin{aligned} S_* &\sim (10^6 \Gamma_0 H_*^2)^{1/(n-2)}, \\ \frac{H_{\text{dec}}}{H_{\text{NR}}} &\sim \frac{10^{-6} \Gamma_0^{2/(n-4)}}{H_*^2}. \end{aligned} \quad (42)$$

The RG induced potential becomes important before inflaton decay if

$$H_{\text{eq}} > H_{\text{pl}}, H_{\text{dec}}, \quad \& \quad h \gtrsim S^{1/2}. \quad (43)$$

The behavior of the zero mode and the fluctuations depends critically on the sign of c_T . The equations of motions are

$$\begin{aligned} \ddot{S} + 3H\dot{S} + AHS^{p-2} \left(p \log\left(\frac{hS}{T}\right) + 1 \right) S &= 0, \\ \ddot{\delta S} + 3H\dot{\delta S} + AHS^{p-2} \left(2p - 1 + p(p-1) \log\left(\frac{hS}{T}\right) \right) \delta S &= 0, \end{aligned} \quad (44)$$

with $A = c_T 2h^4 \Gamma_0$, which can have either sign depending on the sign of c_T . For positive sign $c_T > 0$ and $p \geq 2$ the effective potential is up to factors of $\log(S)$ given by $V_{\text{th}} \propto S^p$. The field rolls towards lower values, and f and D are approximately as given in Eqs. (14, 16). The thermal potential shuts off when $hS \lesssim H$, at which point a plasma mass hT is generated. If $H_{\text{pl}} < H$ the field freezes, and as the temperature drops the plasmons again fall out of equilibrium. The induced RG potential leads to the decrease of S until the plasmons regain equilibrium. And so forth. As a result, the VEV tracks $hS \sim T$. Inflaton decay should happen before the flaton mass or plasma effects become important, i.e., Eqs. (36, 37) should be satisfied. If the tracking is halted due to the effects of the non-renormalizable potential, S decreases further, but the inhomogeneous reheating scenario is still possible for $n > p+2$. Both the effects of V_{NR} and δV with $c_T > 0$ lead to $fD < 1$ and all bounds are stronger compared to case 2.

The situation is completely different for $c_T < 0$. Now the potential is minimized for large S . When damping is negligible, for $p \geq 2$ the instability leads to exponential growth of the zero mode. For $p \geq 3$ the growth is stopped by damping, when $H_{\text{RG}} \sim H^2$, and the VEV tracks $S^{p-2} \sim H/A$. For $p = 2$ however, the thermal mass is up to log factors independent of S and damping remains unimportant. The growth of the zero mode is halted instead when non-renormalizable terms become important, or when the inflaton decays, whatever comes first. If the log term in Eq. (44) does not already dominate initially, with the fast growth of S it will soon do so. Then for $p = 2$ the zero mode and the fluctuations roll in the same effective potential and their ratio remains constant, $D \sim 1$. Since $fD \gg 1$, inflaton decay is earlier making constraints weaker. Moreover, since S increases, the out of equilibrium condition $hS > T$ remains valid.

For an approximately constant decay rate of the form Eq. (2) the time evolution of the flaton will not alter the time of decay. Inflaton decay can still occur, for example, after the flaton starts oscillating in its quadratic potential.

For the same Hubble constant during inflation, the flat direction VEV at inflaton decay is smaller than in case 1-2, due to both the damping of the fluctuations and the evolution of S , see Eq. (19). The decay rate does not alter, but the bounds on H_* , S_* and M to get the right density perturbation are stronger for $f^q D < 1$. Specifically, taking Gaussianity and the requirement $S < M$ into account, constrains $f^q D \gtrsim 10^{-2}$. For oscillations set off by the mass term, the non-renormalizable potential, or by plasma masses $f^q D < 1$. From the expressions for f and D , Eqs. (14, 16, 26), it follows that

$$m_S, H_{\text{pl}} \lesssim 10^2 \Gamma_\phi \quad \& \quad H_{\text{NR}} \lesssim 10^4 \Gamma_\phi \quad (45)$$

for the varying decay rate scenario to work. The flaton VEV and H_* should be tuned even more than for cases 1 and 2.

Things may be different for the RG potential of Eq. (24). For a constant decay rate, the zero mode and fluctuations have mass terms with opposite sign

$$\begin{aligned} \ddot{S} + 3H\dot{S} + \frac{AH}{S^2}S &= 0, \\ \ddot{\delta S} + 3H\dot{\delta S} - \frac{AH}{S^2}\delta S &= 0 \end{aligned} \quad (46)$$

with $A = c_T 2h^4 \Gamma_0$. For $c_T > 0$ the zero mode decreases, while the fluctuations increase, and vice versa for $c_T < 0$. For $c_T > 0$, the decrease of S is halted when the plasmons fall out of equilibrium, and hS tracks T . The interesting case is $q = 1$, since only then $f^q D > 1$. For $c_T > 0$ the growth of the zero-mode is halted by damping, and $\delta V'' \sim H^2$. Only for $q \geq 3$ is $f^q D \propto H^{-(q-2)/2} > 1$.

However, there is not much that can be done to improve the situation. This can easily seen by looking at the expression for the density perturbations at the time of decay, Eq. (9), which implies $\delta S_{\text{dec}} > 10^{-4}M$. Gaussianity and the cutoff constrain $10\delta S_{\text{dec}} < S_{\text{dec}} < M$. For a given cutoff and Hubble scale during inflation, there is only a small window for S_{dec} . For $c_T > 0$, S increases towards earlier time, but it cannot exceed the cutoff scale. For $c_T < 0$, S decreases towards earlier time whereas δS increases, leading to large non-Gaussianities when $S \sim \delta S$. There is no room for S to either decrease or increase much, and a considerable amount of tuning is needed if the RG potential is to dominate at some point.

5 Inflaton decay through non-renormalizable operators

A polynomial decay rate, as in Eq. (1), can arise from non-renormalizable superpotential couplings of the form [4]

$$W = \lambda_0 \frac{q_i}{M} \phi q_j q_k \quad (47)$$

with M the cutoff scale, and $S = \langle q_i \rangle \neq 0$ is the field responsible for the density perturbations. This corresponds to 5-dim non-renormalizable operators in the potential. Such operators may be obtained from integrating out physics above the cutoff scale M . For inflaton decay into MSSM degrees of freedom the combination $q_i q_j q_k$ should form a gauge invariant. For S part of a MSSM flat direction, the only combinations are qqq , $\bar{q}\bar{q}\bar{q}$ and $hq\bar{q}$ with q and \bar{q} left-handed quark/lepton superfields and their charge conjugate. Inflaton decay is unsuppressed for $m_\phi \gtrsim m_q$, with m_q the lightest quark/lepton fields in the superfields q_j, q_k . For S a SM singlet, the only possibility is $Sh_u h_d$ ⁶

The effective coupling is $\lambda = \lambda_0(S/M)$. Then the decay rate is of the form Eq. (1) with⁷

$$\Gamma_0 = \frac{\lambda_0^2}{8\pi M^2} m_\phi, \quad p = 2. \quad (48)$$

with $m_\phi = \beta H_* (\beta\sqrt{H_*})$ for 1FI (2FI), and $\beta \lesssim 1$. The value Γ_0 is bounded from above by λ_0 , $\beta \lesssim 1$. The lower bound comes from the requirement that the inflaton decays before BBN:

$$\Gamma_0 \gtrsim \frac{10^{-47}}{(H_* f D\alpha)^2}. \quad (49)$$

5.1 Low scale inflation

Is the fluctuating decay scenario with a decay rate of the form Eq. (1, 48) compatible with low scale inflation $H_* \sim m_{3/2} \sim 10^{-16}$? And if so, what are the bounds on the couplings and mass of the flaton field?

The decay rate depends on the specifics of the inflaton sector. However, Γ_0 can be bounded by BBN and the scale of inflation to lie in the range $10^{-17}/(f D\alpha)^2 \lesssim \Gamma_0 \sim 10^{-18}(10^{-10})\beta\lambda_0^2/M^2$ for 1FI (2FI). For $M \sim 1$ there is no parameter space for 1FI. The cutoff should be larger than the flat direction VEV $M > S_* \approx 10^{-12}\alpha D H_*$. The gravitino problem is absent for $\Gamma_0 \lesssim 10^7/(f D)^2$.

Case 1 Prompt decay is possible for $10^{10} \lesssim \Gamma_0 \lesssim 10^{19}$, as follows from Eqs. (34). Such large decay rates are not possible in 1FI, while for 2FI it can be obtained for $10^{20} < \beta\lambda_0^2/M^2 < 10^{29}$. This means $10^{-14} \lesssim M \lesssim 10^{-10}$ and $\lambda_0^2\beta \gtrsim 10^{-4}$. The scale M cannot be identified with the scale of inflation, and a new scale has to be introduced in the theory.

The flat direction couplings are unconstrained, whereas its mass $m_S \lesssim 10^{-17}$, in order to obtain a scale invariant spectrum. The effective mass generated by the non-renormalizable potential is also sufficiently small for Planck suppressed operators with $\kappa, \Lambda \sim 1$. Note however, that if these operators are suppressed by the same

⁶In this case a normalizable superpotential coupling $W = \lambda' \phi q_j q_k$ is also possible. The decay through non-renormalizable operators dominates for $\lambda_0 S/M > \lambda'$.

⁷Higher values of p can be obtained through interactions $W = \lambda_0(S/M)^m \phi q_j q_k$ with $m \geq 2$. Since the constraints are stronger for higher values of p we will not discuss this possibility.

scale as the effective inflation coupling, i.e., $\Lambda \sim M$, then $n = 4$ operators should be absent or suppressed $\kappa \ll 1$. Further, $M \gtrsim 10^{-11} (10^{-12})$ for $n = 5 (n = 6)$ operators.

The gravitino problem has to be addressed.

Case 2 For smaller decay rates, $\Gamma_0 \lesssim 10^{10} D^2$, the inflaton decays well after the end of inflation and case 2 and 3 apply. There are additional constraints, of which the strongest ones comes from finite temperature effects: early thermally induced oscillations and thermal evaporation. The inflaton decays while the flat direction field is still frozen, and case 2 applies, if these thermal effects are negligible small.

A word of caution here. All estimates are order of magnitude estimates. But this is even more so when thermal effects are considered. The reason is that the thermally induced mass by particles in (out of) equilibrium is a good approximation in the limit $hS \ll T (hS \gg T)$. However, in the limit $hS \rightarrow T$ both approximations break down.

The first possibility for thermal effects to be absent is that the particles coupling to S fall out of equilibrium before plasma effects become important, and with the RG potential small Eq. (38). This is only possible for small rates Γ_0 , inconsistent with the BBN bound.

The second possibility is that the plasmons are in equilibrium until inflaton decay, and the plasma mass is small, Eq. (37). This requires $h \lesssim 10^{-13} \Gamma_0^{1/2}$. Taking $M \sim 1$, this implies very small couplings and mass: $h \lesssim 10^{-17}$ and $m_S \lesssim 10^{-36}$ for 2FI. Identifying the cutoff with the scale of inflation $M \sim \sqrt{H_*} \sim 10^{-8}$ improves the situation considerably, but still $h \lesssim 10^{-19} (10^{-16})$ and $m_S \lesssim 10^{-20} (10^{-28})$ for 2FI (1FI). The constraints are weakest in the limit $M \rightarrow 10^{-13}$, but at the expense of introducing a new scale in the system, and of gravitino over production.

Finally, there is the possibility that the initial plasma is far from an equilibrium distribution, and the onset of thermal effects is delayed, Eq. (39). $H_{\text{del}} \lesssim \Gamma_\phi$ gives the same constraints as in the previous paragraph, with the replacement $h \rightarrow 10^{-2} h_{\text{pl}}^2$. Here h_{pl} is the coupling of the χ particles (those particles coupling to S) to the plasma. If χ has gauge charges $h_{\text{pl}} \sim 0.1$, and delay of thermal effects is not possible.

The non-renormalizable operators are sub-dominant until inflaton decay, and Eq. (36) is satisfied, for $\Gamma_0 > 10^{-13(n-4)} (\kappa/\Lambda)^{n-3}$. For $M, \Lambda \sim 1$ and 2FI $n = 4$ operators should be suppressed. For $M \sim 10^{-8}$ and $\Lambda \sim 1$ non-renormalizable operators are consistent with 2FI, and with 1FI for $n \geq 5$. However, if $M \sim \Lambda \sim 10^{-8}$ one needs $n \geq 5 (n \geq 6)$ for 2FI (1FI).

Case 3 Case 3 applies if the flat direction field starts rolling in its potential before inflaton decay. If the potential is dominated by the mass term m_S , the plasma mass $m_{\text{pl}} \sim hT$, or $n = 4$ non-renormalizable operators the decay rate decreases faster than the Hubble constant, and decay cannot occur. Non-renormalizable operators with $n \geq 5$, and RG induced potentials with $c_T > 0$ have $fD < 1$ due to evolution of

the zero-mode and its fluctuations. The already tight constraints of case 2 become stronger by appropriate factors of fD .

The only possibly interesting case is when the potential is dominated by RG effects with $c_T < 0$, see Eq. (24). Both the zero-mode and its fluctuations grow exponentially, so that $f \gg 1$ and $D \sim 1$, and hence $fD \gg 1$. A RG potential is generated when (see Eq. (43))

$$h \gtrsim \min[10^{13/2} \sqrt{\Gamma_0}, 10^{5/2}(\Gamma_0)^{1/4}], \quad (50)$$

and therefore $\Gamma_0 \lesssim 10^{-10}$ for couplings less than one. Moreover, if $h > S^{1/2} \sim 10^{-13/2}$ the induced thermal mass is larger than the decay width, and the inflaton start rolling in δV_{th} before decay. The small rates Γ_0 required are naturally for $M \sim 1$. A large cutoff has the additional advantage that the zero mode can grow by a huge amount without exceeding the cutoff. Note also, that small Γ_0 is consistent with the BBN bound for $f \gg 1$.

For all Γ_0 values under consideration⁸, the zero-mode starts growing rapidly as soon as $H \sim H_{\text{RG}} \sim \Gamma_0 h^4$, until its growth is halted by either the non-renormalizable potential ($\delta V_{\text{RG}} \sim V_{\text{NR}}$), or inflaton decay ($\Gamma_\phi \sim H$). Approximating the increase to be instantaneous, the maximum value of the zero mode before the non-renormalizable potential becomes important is $S_{\text{max}}^{n-2} \sim H_{\text{RG}} \sim \Gamma_0 h^4$. Decay happens before this maximum value is reached if

$$\Gamma_\phi \gtrsim H_{\text{RG}} \implies S_{\text{max}} \gtrsim h^2 \quad (51)$$

with $\Gamma_\phi = \Gamma_0 S_{\text{max}}^2$. Eq. (50) implies large couplings, whereas Eq. (51) is satisfied more easily for small couplings. This contradiction can only be remedied for large n and small decay rates. Decay before V_{NR} becomes important does not happen for $n = 4, 5$ operators; it is marginally consistent with $n = 6$ operators in the limit $\Gamma_0 \rightarrow 10^{-26}$ and $h \rightarrow 10^{-13/2}$. The flaton mass has to be extremely small: $m_S \lesssim \Gamma_\phi \sim H_{\text{RG}} \sim 10^{-36}$.

One could contemplate the possibility that the growth of S is halted by non-renormalizable operators, and the flaton slow rolls in the non-renormalizable potential before decay. But this is to no avail. In this case the left hand side of Eq. (51) is $\Gamma_\phi \sim (fD)_{\text{NR}}^2 S_{\text{max}}^2$, with $(fD)_{\text{NR}} \propto H$ parameterizing the evolution of S and δS in the non-renormalizable potential. The right hand side is likewise proportional to H . Thus for $H < H_{\text{NR}}$, both sides decrease with the same rate, and it remains impossible to satisfy Eq. (51). In other words, if decay does not happen before the non-renormalizable potential becomes important, it will certainly not happen during the slow roll in the non-renormalizable potential if the right amount of density perturbations are to be produced.

⁸For $10^{-16} < \Gamma_0 < 10^{-10}$ the growth of the zero mode can start immediately at the end of inflation, at $H \sim H_{\text{end}}$. The non-renormalizable potential becomes important before decay for all n .

5.2 MSSM flat directions

Can the flat directions of the MSSM play the rôle of S for a decay rate of the form Eq. (48)? We take $m_S \sim m_{3/2} \sim 10^{-16}$. The Yukawa couplings of the MSSM vary between 10^{-6} and 1, whereas the gauge couplings $h \sim 0.1$. As discussed in the previous section, only if the inflaton decays promptly at the end of inflation, is low scale inflation consistent with MSSM flatons. But what are the conditions for an arbitrary scale of inflation in the range $10m_S < H_* < 10^{-6}$?

Since the inflaton decay rate has to be greater than the flat direction mass there is always a gravitino problem.

Case 1 Prompt decay occurs for $10^{-6} < \Gamma_0 H_* < 10^3$, see Eq. (34). This is not possible for 1FI for any scale of inflation. For 2FI prompt decay is possible for $H_* \gtrsim 10m_S$, $M \lesssim 10^2 H_*^{3/4} < 1$, and $S_* < M$, which is hardest to satisfy in the limit $H_* \rightarrow H_{\max} \sim 10^{-6}$.

Low scale inflation, with the Hubble constant of the order of the gravitino mass, comes at the cost of introducing a new scale M which cannot be identified with any of the known scales, such as the SUSY breaking scale, the GUT scale, or the Planck scale. The scale M can be identified with the scale of inflation, $M \sim \sqrt{H_*}$ for $H_* \gtrsim 10^{-8}/(\beta^2 \lambda_0^4)$.

Case 2 For smaller Γ_0 the inflaton decays well after the end of inflation, and cases 2 and 3 apply. The decay rate is greater than the MSSM soft mass for $H_* \gtrsim 10^{-10}(fD)^{-1}\Gamma_0^{-1/2}$. Requiring $H_* < 10^{-6}$ and decay after the end of inflation constrains $10^{-4}H_*/(fD)^2 \lesssim \Gamma_0 \lesssim 10^{-6}/(D^2 H_*)$, where we have taken $\alpha \approx 1/6$.

Let's first consider case 2. There are three ways to avoid thermally induced early oscillations. The first possibility is that the plasmons coupling to S fall out of equilibrium before plasma effects become important, and the non-equilibrium thermal mass is small Eq. (38). However, for MSSM gauge couplings the non-equilibrium thermal mass is never small, and this possibility is excluded.

The second possibility for the thermal effects to be negligible is that the plasmons are in equilibrium until inflaton decay, and the plasma mass is small, Eq. (37). Plasma masses are negligible for $h \lesssim \sqrt{H_*}$, which excludes MSSM gauge couplings.

The third possibility is that the thermal plasma is initially far from an equilibrium distribution. However, for gauge particles the thermal effects can be delayed at most until $H_{\text{del}} \gtrsim 0.1H_*$. This offers not much perspective either.

Case 3 Just as for low scale inflation, all thermal constraints are stronger for case 3 if $fD < 1$. Hence, case 3 is likewise incompatible with MSSM flatons. The only possible exception are MSSM flat directions whose potential is dominated by a tachyonic RG mass, i.e., Eq. (24) with $c_T < 0$, so that $fD > 1$ is possible. The zero-mode grows exponentially once the tachyonic mass exceeds the Hubble constant. As discussed in the previous subsection, the inhomogeneous reheating scenario can only

work if the inflaton decays before the growth is halted by the non-renormalizable potential, and Eq. (51) is satisfied.

Oscillations induced by the plasma mass reduce the flaton amplitude. As a result $H_{\text{eq}} \propto H^{14/5}$, and the plasmons remain in equilibrium until decay. The only way out is that (some of the) the particles coupling to S are out of equilibrium from the start $H_{\text{eq}} \gtrsim H_{\text{end}}$, or

$$\Gamma_0 \lesssim 10^6 h^4 H_* . \quad (52)$$

The MSSM flaton couples to several fields. If one of these couplings h' is small so that the above equation is not satisfied, then in addition $H_{\text{RG}} > H_*$ and $H_{\text{RG}} > H_{\text{pl}}$ for the RG induced mass to dominate over the plasma mass from the start. This is only possible for flat direction with top Yukawa interactions $h \sim 1$; in all other cases Eq (52) should be satisfied for all flaton couplings.

Consider first MSSM flat directions with top Yukawas. The RG induced potential can dominate over all other contributions to the potential immediately after the end of inflation. The zero mode and fluctuations grow exponentially. Decay happens before the non-renormalizable potential becomes important for $H_* \lesssim \Gamma_0 \lesssim 10^6 H_*$ and $\Gamma_0 \gtrsim 10^{-9}, 10^{-12}$ for $n = 5, 6$ operators; it is not possible for $n = 4$ operators.

For flat directions without top Yukawa couplings the growth of the zero-mode starts at $H_{\text{RG}} \sim h^4 \Gamma_0 \lesssim H_*$. Moreover, Eq. (52) should be satisfied. These constraints together are severe, prohibiting decay before V_{NR} becomes important for $n \leq 5$, whereas it is only marginally allowed for $n = 6$ in the limit $\Gamma_0 \sim H_0 \rightarrow 10^{-6}$.

6 Constant decay rate

Renormalizable couplings Consider a superpotential coupling of the form [4]

$$W = \lambda \phi H_u H_d \quad (53)$$

with the coupling

$$\lambda = \lambda_0 \left[1 + \left(\frac{S}{M} \right)^q + \dots \right] \quad (54)$$

with the ellipses denoting higher order terms. Since the higher order corrections play no rôle we will omit them in the following. In supersymmetric and string inspired models, all couplings and masses are, rather than being constants, functions of scalar fields in the theory. The higher order corrections can also arise from non-renormalizable operators, such as those in Eq. (47). Within the MSSM the above coupling is the only gauge invariant possibility.

The decay rate is

$$\Gamma_\phi = \Gamma_0 \left[1 + \left(\frac{S}{M} \right)^q \right]^2 , \quad \Gamma_0 = \frac{\lambda_0^2}{8\pi} m_\phi . \quad (55)$$

which is of the form Eq. (2) with $q \geq 1$, $r = 2$ and M some cutoff scale. As before $m_\phi = \beta H_* (\beta \sqrt{H_*})$ for 1FI (2FI) with $\beta \lesssim 1$. The decay rate is bounded from below by BBN

$$\Gamma_0 \gtrsim 10^{-43} \quad (56)$$

In addition the mass of the inflaton should be larger than that of the decay products, otherwise decay is kinematically forbidden.

The density perturbations are given by Eq. (19). All scales in the problem have to lie close together, $H_* \lesssim 10^{-4} M / (\alpha f^q D)$ and $H_* < S_* < M$. The amount of fine-tuning is increased by inefficiencies, when $\alpha f^q D < 1/6$. Considering the density perturbations in terms of the variable at the time of decay, Eq. (9), similarly leads to the conclusion that the amount of fine tuning is increased also in the opposite limit $f^q D > 1$.

Phase space factor Consider 2-body decay of the inflaton, through a coupling of the form

$$W = \lambda_0 \phi \psi \psi \quad (57)$$

The decay rate is

$$\Gamma_\phi = \Gamma_0 \sqrt{1 - \left(\frac{2m_\psi}{m_\phi} \right)^2}, \quad \Gamma_0 = \frac{\lambda_0^2}{8\pi} m_\phi, \quad (58)$$

where the square root comes from integration over phase space. We will refer to this decay rate as the phase space (PS) decay rate. If the mass of ψ is set through a coupling to a flat direction, i.e., $m_\psi = \lambda S$, the decay rate is of the form Eq. (2) with $q = 2$, $r = 1/2$ and $M = m_\psi/(2h)$. Now M is not a fundamental scale in the problem. The density perturbations are given by Eq. (19). Also in this case $M \sim 10 - 10^3 H_*$. This gives the relation

$$\beta \sim \begin{cases} 10 - 10^3 \lambda, & (1\text{FI}), \\ 10 - 10^3 \lambda \sqrt{H_*}, & (2\text{FI}). \end{cases} \quad (59)$$

The lower bound on Γ_0 comes from BBN, Eq. (56). There is also an upper bound, from $\lambda_0 < 1$ and $\beta \lesssim 1 (10^2 \sqrt{H_*})$ for 1FI (2FI).

Note that the back reaction of ψ on the flat direction, induces a thermal mass for S . Said in another way, the couplings of the flaton field to which we refer as h , and which control the strength of the thermal effects, include also λ .

6.1 Low scale inflation

Can the inhomogeneous reheating scenario work for low scale inflation $H_* \sim m_{3/2} \sim 10^{-16}$ for decay rates of the form Eqs. (2, 55, 58)? To obtain the right density fluctuations requires $S_* \sim 1 - 10^2 H_*$ and $M \sim 10 - 10^3 H_*$ for all examples considered above. The less efficient the mechanism — $\alpha(S_*/M)^q$ small — the closer the scales

lie together. M cannot be identified with inflationary scale, and thus introduces a new scale in the problem. This is not the case for the PS decay rate, where M is related to the effective mass of the inflaton decay products, and does not represent a fundamental scale.

The decay rate is bounded by the BBN constraint and the scale of inflation $10^{-43} \lesssim \Gamma_0 \sim 10^{-18}(10^{-10})\lambda_0^2\beta$ for 1FI (2FI). For the PS decay rate $\beta < \beta_{\max} \sim 10^{-6}$ for 2FI as follows from Eq. (59). There is a gravitino problem for $\Gamma_0 \gtrsim 10^{-19}$.

Case 1 Prompt decay is possible for $10^{-16} < \Gamma_0 < 10^{-13}(H_*/M)^q$. Small values for q are favored. Note that for the upper limit, which saturates the Gaussianity constraint, $H_* \sim S_*$ and thus $H_*/M \lesssim 0.1$. Prompt decay is not possible for 1FI, whereas for 2FI it constrains $10^{-6} \lesssim \beta\lambda_0^2 \lesssim 10^{-3}(H_*/M)^q$. It is also incompatible with a PS decay rate.

The window of allowed inflaton mass and couplings is much smaller than for the polynomial decay rate. One reason is that the perturbations are transferred less efficiently to the radiation bath, due to the $(S_*/M)^q$ suppression factors. The second reason is that the constraint $1 \lesssim \Gamma_\phi/H \lesssim 10^3 H_*$ confines $\beta\lambda_0$ also within three decades. This is in contrast with the polynomial decay rate, where due to the S , and thus α , dependence of Γ_ϕ , the parameter combination $\beta\lambda_0/M^2$ is constraint only within nine decades.

The other flat direction couplings are unconstrained, whereas its mass $m_S \lesssim 10^{-17}$ for a scale invariant spectrum. The effective mass from the non-renormalizable potential is also sufficiently small for Planck suppressed operators with $\kappa, \Lambda \sim 1$. However, if these operators are generated at the same scale as the effective cutoff, $\Lambda \sim M$, then the non-renormalizable operators are only marginally consistent for all n .

The gravitino problem has to be addressed.

Case 2 For smaller decay rates, $\Gamma_0 < 10^{-16}$, decay occurs well after the end of inflation, which brings us to case 2 and 3. Non-renormalizable operators play no rôle for $\Gamma_0 > 10^{-28}\kappa/\Lambda, 10^{-43}\kappa/\Lambda^2, 10^{-56}\kappa/\Lambda^3$ for $n = 4, 5, 6$. If $\kappa, \Lambda \sim 1$, then only $n = 4$ operators need to be considered. Non-renormalizable operators with $\Lambda \sim M$ always dominate before decay if $\Gamma_0 < 10^{-16}$.

Let's consider case 2 first. The thermal constraints are severe. They can be satisfied if the particles coupling to S are out of equilibrium, while the non-equilibrium thermal potential remains unimportant, see Eq. (38). The non-equilibrium potential is only small for couplings $\lambda, h \lesssim 10^{-7}$. But such small couplings are inconsistent with the plasmons being out of equilibrium.

The second way in which thermal effects can be avoided, is that the plasmons are in equilibrium, and the plasma mass is small, see Eq. (37). The plasma mass is only negligible for couplings $\lambda, h \lesssim \sqrt{\Gamma_0}$. The couplings scale with $\sqrt{\Gamma_0}$, and must be especially small for 1FI. The flaton mass is bounded by $m_S \lesssim \Gamma_0$. The constraints

are weakest in the limit $\Gamma_0 \rightarrow 10^{-16}$, i.e., when decay happens shortly after the end of inflation.

Finally, the thermal effects are delayed if the initial plasma is far from an equilibrium distribution. The ensuing constraints are the same as in the previous paragraph with the replacement $h \rightarrow 10^{-2}h_{\text{pl}}^2$. The plasma couplings have to be small $h_{\text{pl}} \lesssim 10^{-3}(\Gamma_0/10^{-16})^{1/4}$.

Case 3 The non-renormalizable potential generically plays no rôle. If either the zero temperature mass or the plasma mass becomes important before inflaton decay, this implies fine-tuning the already tuned values of H_* , S_* , M . And without opening parameter space much, see Eq. (45).

The RG potential offers no better prospects, whether c_T is positive or negative. The VEV S is bounded by the scale of inflation H_* and the cutoff M , both at the end of inflation and at the time of inflaton decay. To obtain the observed density perturbations, all scales have to lie within four decades of each other at all times, and consequently there is not much room for evolution of S and δS .

6.2 MSSM flat direction

Can the MSSM flat directions be responsible for the density fluctuations, for a constant decay rate of the form Eqs. (2, 55, 58)? MSSM scalars have soft SUSY breaking mass $m_S \sim 10^{-16}$, and at least one or more gauge couplings with $h \sim 0.1$. What are the conditions on H_* and Γ_0 for a successful scenario?

Case 1 Just as for low scale inflation, prompt decay is only possible for a 2FI within a small window $10^2\sqrt{H_*} \lesssim \beta\lambda_0^2 \lesssim 10^5(H_*/M)^q$. The scale of inflation can be identified with the cutoff scale only for $H_* \sim 10^{-6}$ and $M \sim \sqrt{H_*} \sim 10^{-3}$. In this case the inflaton mass and coupling have to be large $\beta\lambda_0^2 \rightarrow 1$. Prompt decay is incompatible with a PS decay rate for all scales of inflation.

The flaton coupling and mass are unconstrained, except for $m_S \lesssim 0.1H_*$ from scale invariance. The effective mass generated by non-renormalizable operators is likewise sufficiently small for cutoffs $\Lambda > M$.

The gravitino problem has to be addressed.

Case 2 There are two possibilities for thermal effects to be absent. Either the induced plasma mass is negligible, or if the plasmons are out of equilibrium, the induced RG mass is sub-dominant, Eqs. (37, 38). This is the case for $h \lesssim \sqrt{\Gamma_0}$, respectively $h \lesssim \sqrt{S_*} \sim 10\sqrt{H_*}$. Since $H_* \lesssim 10^{-6}$, and $\Gamma_0 \lesssim H_*$ (otherwise prompt decay), both options are inconsistent with gauge couplings of order $h \sim 0.1$.

The onset of thermal effects can be delayed, see Eq. (39), only for $H_* \rightarrow 10^{-6}$, and even then $\Gamma_\phi \gtrsim 0.1H_*$. Hence, this hardly opens up parameter space.

Case 3 If decay happens well after the end of inflation, thermal effects are always important. Decay should follow quickly after the onset of thermally induced motion of S and δS . This is because Gaussianity and the cutoff constrain δS , S , and M to lie close to each other both at the end of inflation and at the time of decay, and there is little room for evolution of S and δS . Therefore, $H_{\text{th}} \lesssim 10^2 \Gamma_\phi$ or equivalently $\Gamma_0 \gtrsim 10^{-2} H_*$. Only a small bit of parameter space is opened up, and only at the cost of fine tuning all parameters.

7 Varying mass scenario

Suppose the field dominating the energy density has an effective mass set by a coupling to a flat direction $m_\psi \sim \lambda S$, with λ a gauge/Yukawa coupling. If the ψ field couples to SM degrees of freedom with a coupling λ_0 the decay width is

$$\Gamma_\phi \sim \frac{\lambda_0^2}{8\pi} \lambda S. \quad (60)$$

Decay is unsuppressed if m_ψ is larger than the masses of the decay products. The decay width is polynomial in S , of the form Eq. (1), with

$$\Gamma_0 = \frac{\lambda \lambda_0^2}{8\pi}, \quad p = 1. \quad (61)$$

The observed perturbations are obtained, if Eq. (18) is satisfied with $p = 1$. The decay rate decreases with S , and $m_S < \Gamma_\psi$ for decay to take place at all. The thermal bath originates from inflaton decay, and thus the thermal time scales are as given in Eqs. (29, 30, 31) with Γ_ϕ the S -independent inflaton decay rate.

There are important differences with the examples discussed up till now, in which the inflaton was the FDED. Now, a new field is introduced in the theory, that is to dominate the energy density sometime after inflaton decay. The new field is accompanied by new parameters, making the model less predictive. There is no equivalent of case 1, as the inhomogeneous reheating mechanism only works if ψ decays after it (nearly) dominates the energy density, $\Gamma_\psi < H_{\text{dom}}$. The thermal bath is produced by inflaton decay, and in the period between $H_{\text{end}} < H < \Gamma_\psi$, thermal effects should be taken into account. Thus, in contrast with the inflaton as FDED, a thermal bath is already present *before* ψ domination.

7.1 Constraints

Here we discuss the conditions specific to the varying mass scenario, namely the requirement of domination, and the thermal effects before domination.

Consider first the case in which the ψ quanta are initially in thermal equilibrium. Further assume that the annihilation rate is smaller than $\Gamma_{\text{ann}} \lesssim m_\psi^2$.⁹ Then

⁹For larger annihilation rates freeze-out occurs at lower temperature $T \sim m_\psi/20$, and the

the ψ particles come to dominate the energy density as soon as they become non-relativistic, at $T_{\text{dom}} \sim m_\psi$ or $H_{\text{dom}} \sim m_\psi^2$. As was first noted in [7], the ψ plasmons induce a thermal mass for the flat direction field S of the form $m_S \sim \lambda T$. Requiring this thermal mass to be sub-dominant at all times gives

$$m_{\text{th}} \lesssim H_{\text{dec}} < H_{\text{dom}} \Rightarrow S > M_{\text{pl}} \quad (62)$$

Such large VEVs are hard to reconcile with low scale inflation $H_* \lesssim H_{\text{max}}$, and with the presence of non-renormalizable operators. What is more, for an initial trans-Planckian VEV the flat direction field itself will come to dominate the energy density of the universe while still frozen, leading to an S -dominated period of inflation.

The assumptions made in the above argument is that the ψ -quanta reach thermal equilibrium before domination, and that the FDED decays while S is still frozen. If either one of these assumptions is dropped, the negative conclusion implied by Eq. (62) may be avoided. We discuss some possibilities in turn.

A) One possibility is that the ψ quanta are direct inflaton decay products, which are out of equilibrium from the beginning

$$m_\psi \gtrsim T \sim \sqrt{\Gamma_\phi} \quad (63)$$

This requires large inflaton masses $m_\phi > m_\psi$. The initial distribution is non-thermal, and n_ψ/s will remain constant if in addition the annihilation rate small. This means sufficiently small ψ couplings, ruling out the possibility of identifying ψ with an MSSM field. Moreover the couplings between ψ and the MSSM sector have to be sufficiently small so that ψ decays after domination:

$$\Gamma_\psi < H_{\text{dom}}. \quad (64)$$

An RG potential of the form Eq. (24) is induced, which is negligible only for $H_{\text{RG}} < \Gamma_\psi$ or

$$h \lesssim \sqrt{S} \left(\frac{\Gamma_\psi}{\Gamma_\phi} \right)^{1/4}. \quad (65)$$

Once again small couplings are needed, ruling out the possibility of identifying S with a MSSM flat direction. Otherwise, if the RG potential does get important and $c_T > 0$, this will lead to damping, leading to non-Gaussianity for $D \lesssim 10^{-3}$. For $c_T < 0$ on the other hand, S decreases until the ψ -quanta acquire thermal equilibrium. This brings us straight back to the constraint Eq. (62).

To summarize, small couplings are needed for the annihilation rate and the induced thermal mass to be small, and MSSM scalars cannot play a rôle. There is a tension with the out of equilibrium condition, which requires a large coupling.

B) The ψ field dominates the energy density immediately after the end of inflation. This requires the inflaton to decay (almost) exclusively into ψ quanta. Note number density n_ψ is Boltzmann suppressed. Domination then happens at Hubble constants $H_D \ll m_\psi$ and the constraint in Eq. (62) becomes much stronger

that although Eq. (62) is trivially avoided in this way, soon after the end of inflation either the plasma mass or the RG potential generated by the coupling $W = \lambda S\psi\psi$ will become important. The situation is the same as for a polynomial decay rate, as discussed in section V, except that case 1 is excluded.

C) The inflaton decays non-perturbatively, through resonance effects. The initial distribution of ψ particles is far from equilibrium, as well as all the other particles in the universe. The highly non-linear, non-equilibrium character of preheating makes it impossible to make any definite predictions.

D) The rate of number changing interactions is small, and chemical equilibrium is not attained. The initial distribution is non-thermal if the ψ 's are direct inflaton decay products. The plasma mass $m_{\text{pl}} \sim hn_\psi/E_\psi$ can be smaller than its equilibrium value. However, to avoid that $H_{\text{dom}} \propto n_\psi E_\psi$ is likewise smaller, and Eq. (62) is not ameliorated, it should be E_ψ exceeding its equilibrium value rather than n_ψ being below its equilibrium value.

E) The thermal mass $m_{\text{pl}} \sim \lambda T$ does become important before inflaton decay. The flaton field starts oscillating in the potential well with decreasing amplitude $S \propto H^{7/9}$, see Eq. (26). If ψ decays during the thermally induced oscillations, i.e., before the zero temperature potential becomes important, then $f \sim (\Gamma_\psi(S_*)/H_{\text{pl}})^{7/2}$ and $D \sim 1$. Unless $\Gamma_\psi(S_*) \rightarrow H_{\text{pl}}$ decay is delayed, and the thermal constraints, as well as the constraint $m_\psi < \Gamma_\psi$, are much stronger.

7.2 Model building

The varying mass scenario, in which the FDED is not the inflaton but some other field, is more elaborate than scenarios in which the FDED is the inflaton. There is an extra step: the production and subsequent domination of ψ quanta. Not only does the introduction of an extra field lead to more parameters, and thus to less predictability, it also introduces extra constraints. In particular, there is already a thermal plasma before the domination of ψ , and thermal effects should be taken into account.

If the initial distribution of χ quanta is an equilibrium one, the scenario does not work, because of Eq. (62). The natural way out is to assume the ψ quanta never are in thermal equilibrium. However, non-equilibrium thermal effects still play a role, and it requires small coupling and/or tuning to make it work.

The fluctuating mass scenario is not advantageous for low scale inflation. Prompt decay at the end of inflation is not possible, as decay should occur only after the ψ -quanta come to dominate the universe. Late decay implies strong thermal constraints, and only very small flaton masses and couplings are consistent. The prospects are much worse than for the inflaton as FDED.

Thermal constraints, which already play a role before domination, can generically only be avoided for sufficiently small couplings. This excludes the possibility of identifying ψ with an MSSM scalar, as well as the possibility of identifying the flaton S with an MSSM flat direction.

8 Conclusions

In the inhomogeneous reheating scenario not the inflaton but some other field is responsible for the observed density perturbations. It would be economical if the new fields and scales introduced in this scenario could be identified with the fields and scales appearing in our models of particle physics. In particular, we have addressed the following two questions. Is low scale inflation possible, with the Hubble scale of the order of the gravitino, such that the inflaton sector can be naturally identified with the SUSY breaking sector? Can any of the MSSM flat directions be responsible for the density fluctuations?

We discussed various decay rates, obtained from non-renormalizable couplings, renormalizable couplings, and from phase space effects. For the last two examples, the observed density perturbations can only be obtained if the Hubble constant, the flaton VEV, and the cutoff scale all lie within 4 decades. This requires some tuning, especially since Gaussianity of the perturbations is only assured for $S_* \gtrsim 10H_*$. For the phase space decay rate the cutoff is not a fundamental scale in the theory. This has the advantage that there is no need to explain the origin of this scale. The disadvantage is that this model is quite constraint, and for example, prompt decay is excluded. For the non-renormalizable decay rate there is more freedom, since the density perturbations do not depend on the cutoff.

After inflation, already before inflaton decay has completed, there is a dilute plasma. Fields coupling to the flat direction field, whether they are in equilibrium with the thermal bath or not, will induce a thermal mass for S . Such thermal corrections will lead to early induced oscillations of the flat direction field, which are generically fatal for the inhomogeneous reheating scenario. The thermal effects can be avoided if all flat direction couplings are small, which excludes MSSM flat directions. For a polynomial decay rate, which can originate from non-renormalizable couplings, in addition the flaton mass has to be much smaller than the Hubble scale during inflation.

The thermal constraints can be trivially avoided if the inflaton decays promptly at the end of inflation, so that there is no time for thermal effects to act. In this case the flat direction mass and couplings are unconstrained, except for the requirement that $m_S \lesssim 0.1H_*$. The reheat temperature is high, and there is a potential gravitino problem. Prompt decay however, is only consistent with a Gaussian perturbation spectrum for a small window of inflaton masses and couplings. The constraints are stronger for a constant decay rate. In particular, decay rates which inherit their S dependence from phase space effects are incompatible with prompt decay. Large inflaton mass and couplings are needed, inconsistent with one field models of inflation.

Finally, we discussed a variation of the inhomogeneous reheating scenario, in which not the inflaton field but another field has a fluctuating decay rate. After inflation, this field first has to come to dominate the energy density, and then decay. The prospects for model building are bleak. There is no analog of prompt decay:

thermal effects are always there, even before the field comes to dominate the energy density. Only with small couplings can one avoid the disastrous consequences of thermally induced masses. This excludes the possibility of identifying either the decaying field or the flat direction field with an MSSM field. Moreover, there is an extra step in the model, making it less advantageous for low scale inflation.

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